SOL HW 8.2B

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Name: Sol. Date:

Pre Calculus 11: HW Section 8.2b Solving Systems of Equations by Substitution

1. Solve each system by using substitution:

i) $2x+3y=10 \quad y=3x-5$

$2x+3(3x-5)=10$
$2x+9x-15=10$
$11x=25$
$x=\frac{25}{11}$

$y=3\left(\frac{25}{11}\right)-5$
$y=\frac{75}{11}-\frac{55}{11}$
$y=\frac{20}{11}$

$\therefore \left(\frac{25}{11},\frac{20}{11}\right)$

ii) $3x+2y=8 \quad x=12y-10$

$3(12y-10)+2y=8$
$36y-30+2y=8$
$38y=38$
$y=1$

$x=12(1)-10$
$x=2$

$\therefore (2,1)$

iii) $\frac{x}{3}+\frac{y}{2}=\frac{1}{6} \quad x=6y+8$

$\frac{6y+8}{3}+\frac{y}{2}=\frac{1}{6}$
$12y+16+3y=1$
$15y=-15$
$y=-1$

$x=6(-1)+8$
$x=2$

$\therefore (2,-1)$

iv) $2x=10+3y \quad 4(x+y)=42-y$

$4x=20+6y$

$4x+4y=42-y$
$(20+6y)+4y=42-y$
$20+10y=42-y$
$11y=22$
$y=2$

$\therefore 2x=10+3(2)$
$2x=16$
$x=8$

$\therefore (8,2)$

v) $3x+20=3y \quad 9x+32=y$

$3x+20=3(9x+32)$
$3x+20=27x+96$
$-76=24x$
$\frac{-76}{24}=x$
$\frac{-19}{6}=x$

$y=3\left(\frac{-19}{6}\right)+32$
$y=\frac{-57+64}{2}$
$y=\frac{-3}{2}$

$\therefore \left(\frac{-19}{6},\frac{-3}{2}\right)$

vi) $2(4-2x)=26-2y \quad 3(8-3x)=-2(y-7)$

$8-4x-26=-2y$
$-4x-18=-2y$

$-\frac{4(28)}{5}-\frac{5(18)}{5}=-2y$
$\frac{-112-90}{5}=-2y$
$\frac{-202}{5}=-2y$
$\frac{202}{10}=y$
$20.2=y$

$24-9x=(-2y)+14$
$24-9x=(-4x-18)+14$
$24-9x=-4x-4$
$28=5x$
$\frac{28}{5}=x$

$\therefore (5.6, 20.2)$

vii) $y = x - \frac{1}{4} \qquad 0 = x^2 - y$

$0 = x^2 - \left(x - \frac{1}{4}\right)$

$0 = x^2 - x + \frac{1}{4}$

$0 = 4x^2 - 4x + 1$

$0 = (2x-1)^2$

$x = \frac{1}{2}$

$y = \frac{1}{2} - \frac{1}{4}$

$y = \frac{1}{4}$ $\therefore \left(\frac{1}{2}, \frac{1}{4}\right)$

viii) $y = \frac{x}{2} + 4 \qquad y = |x|$

$\frac{x}{2} + 4 = |x|$

$x = \frac{x}{2} + 4$

$\frac{x}{2} = 4$

$x = 8$

$y = \frac{8}{2} + 4$

$y = 8$

$\therefore (8, 8)$

$x = -\frac{x}{2} - 4$

$\frac{3x}{2} = -4$

$x = -\frac{8}{3}$

$y = -\frac{8}{3}\left(\frac{1}{2}\right) + 4$

$y = -\frac{4}{3} + \frac{12}{3}$

$y = \frac{8}{3}$

$\therefore \left(-\frac{8}{3}, \frac{8}{3}\right)$

ix) $y = -(x-2)^2 + 2 \qquad y = x^2$

$x^2 = -(x-2)^2 + 2$

$x^2 = -(x^2 - 4x + 4) + 2$

$x^2 = -x^2 + 4x - 4 + 2$

$2x^2 - 4x + 2 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)^2 = 0$

$x = 1$

$y = 1$

$\therefore (1, 1)$

x) $y = \frac{4}{x} \qquad y = x - 3$

$x - 3 = \frac{4}{x}$

$x^2 - 3x = 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, \; x = -1.$

$y = 1 \qquad y = -4.$

$\therefore (4, 1) \; ; \; (-1, -4)$

xi) $13(x+y) = 15x - 8 \qquad 3x + 3y = 9(x-2)$

$13x + 13y = 15x - 8$

$13y = 2x - 8$

$13y + 8 = 2x$

$13\left(\frac{5}{6}\right) + 8 = 2x$

$\frac{65}{6} + \frac{48}{6} = 2x$

$\frac{113}{12} = x$

$3x + 3y = 9x - 18$

$3y = 6x - 18$

$y = (2x) - 18$

$y = 13y + 8 - 18$

$0 = 12y - 10$

$10 = 12y$

$\frac{5}{6} = y$

$\therefore \left(\frac{113}{12}, \frac{5}{6}\right)$

xii) $x^2 + 4x + 8 = y^2 \qquad (x+2) = 2y$

② $x^2 + 4x + 8 = \frac{x^2 + 4x + 4}{2}$

$2x^2 + 8x + 16 = x^2 + 4x + 4$

$x^2 + 4x + 12 = 0$

$x = \frac{-4 \pm \sqrt{16 - 4(1)(12)}}{2}$

$x = \frac{-4 \pm \sqrt{16 - 48}}{2}$

$x = \frac{-4 \pm \sqrt{-32}}{2}$ ↖ No Real Soln

① $\frac{x+2}{2} = y$

$\frac{x^2 + 4x + 4}{2} = y^2$

2. If $ax + ay = 4$ and $x + y = 17$, what is the value of "a"?

$a(x+y) = 4$

2. If $ax+ay=4$ and $x+y=17$, what is the value of "a"?

$a(x+y)=4$
$a(17)=4$
$a=\frac{4}{17}$ //

3. Determine all pairs (x,y) that satisfy the system of equations: $x+y=0$ $x^2-y=2$

$y=-x$ $x^2-y=2$ $x=-2$ $x=1$
$x^2-(-x)=2$ $y=2$ $y=-1$
$x^2+x-2=0$ $(-2,2)$ $(1,-1)$
$(x+2)(x-1)=0$

4. If $x-y=4\sqrt{2}$ and $xy=56$, determine the two possible values of "x+y".

$(x-y)^2=16(2)$
$x^2-2xy+y^2=32$
$x^2-2xy+y^2+4xy=32+4(56)$
$x^2+2xy+y^2=256$
$(x+y)^2=256$
$x+y=\pm 16$

5. The line $y=2x+2$ intersects the parabola $y=x^2-3x+c$ at two points. One of these points is (1,4). Determine the coordinates of the second point of intersection. [Euclid]

① $x=1$ $y=4$
$4=1^2-3(1)+c$
$4=1-3+c$
$4=-2+c$
$6=c$

② $2x+2=x^2-3x-6$
$0=x^2-5x-8$
$0=(x-4)(x-1)$
$x=4$ $x=1$
$y=10$ $y=4$.

$\therefore (1,4)$ & $(4,10)$ //

6. If $(x+1)(x-1)=8$, then what is the value of $(x^2+x)(x^2-x)$

① $x^2-1=8$
$x^2=9$.

② $x(x+1)(x)(x-1)=?$
$x^2(x+1)(x-1)=?$
$9(8)=?$
$72=?$

7. Challenge: If "x" and "y" are real numbers, determine all solutions (x,y) of the system of equations:

① $x^2-xy+8=0$
② $x^2-8x+y=0$ → $x^2-8x=-y$
$-x^2+8x=y$

1) Isolate "y" from ②

2) $x^2-x(-x^2+8x)+8=0$
$x^2+x^3-8x^2+8=0$
$x^3+x^2-8x^2+8=0$
$x^2(x+1)-(8x^2-8)=0$
$x^2(x+1)-8(x^2-1)=0$
$x^2(x+1)-8(x+1)(x-1)=0$

$(x+1)[x^2-8(x-1)]=0$
$(x+1)[x^2-8x+8]=0$

$x=-1$ $x=\frac{8\pm\sqrt{64-4(8)}}{2}$

$x=\frac{8\pm\sqrt{32}}{2}$

① $y=-x^2+8x$

$(-1)^2+8(-1)$

① $y = -x^2 + 8x$

$y = -(-1)^2 + 8(-1)$

$y = -1 - 8$

$y = -9.$

$\therefore (-1, -9)$ //

$x = \frac{8 \pm \sqrt{32}}{2}$

$x = 4 \pm 2\sqrt{2}$

② $y = -(4+2\sqrt{2})^2 + 8(4+2\sqrt{2})$

$y = -(16 + 16\sqrt{2} + 8) + 32 + 16\sqrt{2}$

$y = -24 + 32$

$y = 8$

$\therefore (4+2\sqrt{2}, 8)$ //

③ $y = -(4-2\sqrt{2})^2 + 8(4-2\sqrt{2})$

$y = -(16 - 16\sqrt{2} + 8) + 32 - 16\sqrt{2}$

$y = -16 + 16\sqrt{2} - 8 + 32$

$y = 8$

$\therefore (4-2\sqrt{2}, 8)$ //